Being comfortable with common ratios can save you a lot of time on the GMAT. Last week we covered distance/rate problems. Another great application of ratios is work rate problems. An important relation that helps us solve work rate problems is:

Work Done = Rate * Time

This relation will lead a perceptive observer to draw a parallel with another very popular relation most of us have come across:

Distance = Speed * Time

Speed is the same as Rate of work i.e. how fast you cover some distance or how fast you complete some given work. So obviously, if we can use ratios to solve many Distance Speed Time problems, we should be able to solve many Work Rate Time problems using ratios too.

Let's look at some examples. Try and solve each one of them on your own before you go through the solutions.

Example 1:

A tank has 5 inlet pipes. Three pipes are narrow and two are wide. Each of the three narrow pipes works at 1/2 the rate of each of the wide pipes. All the pipes working together will take what fraction of time taken by the two wide pipes working together to fill the tank?

(A) 1/2
(B) 2/3
(C) 3/4
(D) 3/7
(E) 4/7

We are given that rate of work of 1 narrow pipe : rate of work of 1 wide pipe = 1:2

If we can find the ratio of rate of work of 2 wide pipes : rate of work of all pipes together, then we can easily get the ratio of time taken by 2 wide pipes : time taken by all pipes together. This is because ratio of time taken will be inverse of the ratio of rate of work since work done in both the cases is the same. (For a further explanation of this concept, check out the previous post)

In ratio terms, rate of work of 3 narrow pipes is 1*3 and rate of work of 2 wide pipes is 2*2

Therefore, rate of work of 3 narrow pipes : rate of work of 2 wide pipes = 3:4

Or we can say rate of work of 2 wide pipes : rate of work of all pipes together = 4 : (3+4) = 4:7

Then, time taken by 2 wide pipes : time taken by all pipes together = 7:4 (i.e. inverse of 4:7)

So all the pipes together will take 4/7 th of the time taken by the two wide pipes.

Answer (E)

Example 2:

Working at their respective constant rates, Paul, Abdul and Adam alone can finish a certain work in 3, 4, and 5 hours respectively. If all three work together to finish the work, what fraction of the work will be done by Adam?

(A) 1/4
(B) 12/47
(C) 1/3
(D) 5/12
(E) 20/47

It is given that:

Time taken by Paul : Time taken by Abdul : Time taken by Adam = 3:4:5

Rate of work must be inverse of time taken. But how do you take the inverse when you have a ratio of 3 quantities? Does it become 5:4:3? No. Actually it becomes 1/3 : 1/4 : 1/5 (I will explain the 'why' for this when I take variation)

Rate of Paul : Rate of Abdul : Rate of Adam = 1/3 : 1/4 : 1/5

Let's multiply this ratio by the LCM to convert it into integral form. The LCM of 3, 4 and 5 is 60.

Rate of Paul : Rate of Abdul : Rate of Adam = (1/3)*60 : (1/4)*60 : (1/5)*60 = 20:15:12 (I would like to remind you here that multiplying or dividing each term of a ratio by the same number does not alter the ratio)

So if the total work is 20+15+12 = 47 units, Adam will complete 12 units out of it. Hence the fraction of work done by Adam will be 12/47.

Answer (B)

Example 3:

Machines A and B, working together, take t minutes to complete a particular work. Machine A, working alone, takes 64 minutes more than t to complete the same work. Machine B, working alone, takes 25 minutes more than t to complete the same work. What is the ratio of the time taken by machine A to the time taken by machine B to complete this work?

(A) 5:8
(B) 8:5
(C) 25:64
(D) 25:39
(E) 64:25

In this question, you can think logically to arrive at the answer quickly.

When machine A is working alone, it takes 64 extra minutes. Why? Because there is work leftover after t minutes. The work that would have been done by machine B in t minutes is leftover and is done by machine A in 64 minutes.

Time taken by A : Time taken by B = 64:t ….. (I)

Similarly, when machine B works alone, it takes 25 extra minutes to complete the work that machine A would have done in t minutes.

Time taken by A : Time taken by B = t:25 ……(II)

From (I) and (II) above,

$64/t = t/25$

$t = 40$

Time taken by machine A : Time taken by machine B = $t:25 = 40:25 = 8:5$

Answer (B)